

# Parametric Modeling of Low-Frequency Water-Vapor-Induced Tropospheric Path Length Fluctuations

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*Detailed wet tropospheric fluctuation information will be required to support proposals to search for gravitational waves in ultra-precise Doppler data. In this article, similarities between the solar wind and tropospheric effects on apparent signal path length ("signal delay") are used to hypothesize the following parametric model for low-frequency wet tropospheric path length fluctuation (with  $\sigma_{R_w}$  = RMS signal delay fluctuation,  $R_{wz}$  = total zenith signal delay,  $\theta$  = elevation angle, and  $\tau_a$  = averaging time):*

$$\sigma_{R_w}(R_{wz}, \theta, \tau_a) = 2 \times 10^{-2} R_{wz} (\sin \theta)^{-1} (\tau_a/1000)^{0.6}$$

*Recent experimental observations of wet tropospheric signal delay fluctuations can be interpreted as confirming this parametric form. The model is used to suggest the appropriate conditions for collection of experimental tropospheric fluctuation data.*

## I. Introduction

Proposals have recently been advanced to search for "gravitational waves" in ultra-precise two-way Doppler data. Preliminary estimates of gravitational wave characteristics indicate that a total measurement system fractional frequency fluctuation ( $\sigma(\Delta F/F)$ , where  $F$  is an S- or X-band frequency) of  $1 \times 10^{-15}$  over the time scales of interest (50 to 5000 seconds) will be required (Ref. 1). At this level, fluctuations in the wet (water vapor) component of the tropospheric signal delay ( $R_w$ ) will surely constitute a major error source. Unfortunately, little work has been done to date on the (complete) parametric form of the wet tropospheric fluctuation spectrum. This article *hypothesizes* a tropospheric fluctuation model based on observed functional similarities in the tropospheric

and solar wind effects on microwave frequency propagation. The derived model is then utilized to suggest appropriate conditions for the acquisition of experimental tropospheric fluctuation data.

## II. Derivation of a Tropospheric Fluctuation Model

The recent past has seen a considerable effort expended in the parametric modeling of the solar wind fluctuation spectrum (for example, Refs. 2 and 3). In light of functional similarities between the troposphere and solar wind, it seems reasonable to investigate whether this recently acquired knowledge might be useful in understanding and modeling tropo-

spheric fluctuations. Although the solar wind is composed of charged particles, in contrast to the neutral particles of the wet troposphere, there are two very basic similarities between the solar wind and the troposphere:

- (1) The apparent signal path length ("signal delay") increases in proportion to the total columnar particulate matter.
- (2) The particulate matter convects as a "wind" across and along the signal path.

The recently found significant features of the solar wind fluctuation spectrum (for instance, Refs. 2 and 3) are as follows:

- (1) The fluctuations are power law with time scale.
- (2) The fluctuations are proportional to the total particulate density along the signal path.

Experimental evidence that wet tropospheric signal delay fluctuations are power law with time scale has been obtained by Thompson *et al.*, (Ref. 4). Further, Thompson is quoted (Ref. 5) as stating: "The RMS phase fluctuation is proportional to the total path length." Since the test paths utilized by Thompson are line-of-sight between surface (Hawaiian) stations, one expects a nearly constant water vapor density *per unit path length*, and hence a total columnar density nearly proportional to the total path length. Thompson's statement would certainly appear to imply experimental confirmation that wet tropospheric signal delay fluctuations are (approximately) proportional to total (particulate) columnar density.

If one were to use these relationships in hypothesizing a wet tropospheric fluctuation model, the result would be:

$$\sigma_{R_w}(R_w, \tau_a) = K_0 R_w (\tau_a)^{K_1}$$

where

$\sigma_{R_w}$  = RMS wet tropospheric path length fluctuation

$\tau_a$  = averaging time, seconds

$R_w$  = total wet tropospheric path length, centimeters

$K_0, K_1$  = coefficients

One further models (Ref. 6) the wet tropospheric signal delay variation with elevation angle as:

$$R_w \approx R_{wz} (\sin \theta)^{-1}$$

where

$R_{wz}$  = total wet tropospheric path length at zenith, centimeters

$\theta$  = elevation angle

so that one has a parametric form as follows:

$$\sigma_{R_w}(R_{wz}, \theta, \tau_a) = K_0 R_{wz} (\sin \theta)^{-1} (\tau_a)^{K_1}$$

It would now be appropriate to inquire as to expected values for the coefficients  $K_0$  and  $K_1$ . Thompson (Ref. 4) determined a value of -2.6 for the wet tropospheric columnar fluctuation spectral index. This directly yields a value for  $K_1$  of 0.8, which, rather interestingly, happens to be very close to the numerical value for the columnar solar wind fluctuation ( $\sim 0.7$ ). A value for  $K_0$  cannot be obtained directly from Ref. 4 since total wet tropospheric signal delay values are not indicated for the phase spectral density measurements; however, Ref. 7 provides a very tentative estimate of:

$$\sigma_{R_w} \approx 2 \times 10^{-2} (R_w); \quad \tau_a = 1000 \text{ seconds}$$

These estimates for  $K_0$  and  $K_1$  yield:

$$\sigma_{R_w}(R_{wz}, \theta, \tau_a) = 2 \times 10^{-2} R_{wz} (\sin \theta)^{-1} (\tau_a/1000)^{0.8}$$

The model is tested for reasonableness by computing the averaging time for which:

$$\sigma_{R_w}/R_w = 0.5$$

The value of 0.5 ( $\sigma_{R_w}/R_w$ ) is chosen as an upper bound for the very low frequency fluctuations. With this model a value of 0.5 is achieved for averaging times of about 16 hours. This (averaging time) value seems somewhat small; one might expect the lowest frequency fluctuations to be related to the movement of large scale air masses occurring over periods of several days, at least for temperate, semi-arid Deep Space Station (DSS) locations like Goldstone, California (one could expect a substantially different very low frequency wet tropospheric fluctuation spectrum for radically different climatic environments; for example, near ocean, tropical, and polar). To accommodate the expected DSS climatic environment, a value of 0.5 ( $\sigma_{R_w}/R_w$ ) will be adopted for  $\tau_a = 260,000$  seconds (approximately 3.0 days); this yields a somewhat smaller value for  $K_1$  ( $\approx 0.6$ ). The final hypothesized wet tropospheric model then becomes:

$$\sigma_{R_w}(R_{wz}, \theta, \tau_a) = 2 \times 10^{-2} R_{wz} (\sin \theta)^{-1} (\tau_a/1000)^{0.6}$$

### III. Model Results

Two bounding cases are briefly considered as follows:

- (1) Desert winter (cold and dry)

$$R_{wz} \approx 2 \text{ centimeters}$$

- (2) Desert summer (warm and humid)

$$R_{wz} \approx 20 \text{ centimeters}$$

These assumptions translate to fractional frequency fluctuation as follows (with  $\sigma(\Delta F/F) \equiv \sigma_{R_w} / \tau_a \cdot c$ ;  $c$  = speed of light):

$R_{wz} \backslash \tau_a$	100 seconds	1000 seconds
2 centimeters	$3.3 \times 10^{-15} (\sin \theta)^{-1}$	$1.3 \times 10^{-15} (\sin \theta)^{-1}$
20 centimeters	$3.3 \times 10^{-14} (\sin \theta)^{-1}$	$1.3 \times 10^{-14} (\sin \theta)^{-1}$

The above data are graphically illustrated in Fig. 1. The significance of these very preliminary estimates is that for the following conditions:

- (1) Winter climatic conditions
- (2) Longer averaging times
- (3) Near zenith observations

wet tropospheric signal delay fluctuations may not pose too serious an error source for the gravitational wave detection experiment. On the other hand, the following conditions:

- (1) Summer climatic conditions
- (2) Shorter averaging times
- (3) Low elevation observations

will most certainly present a very difficult error source for the gravitational wave detection experiment.

### IV. Discussion and Summary

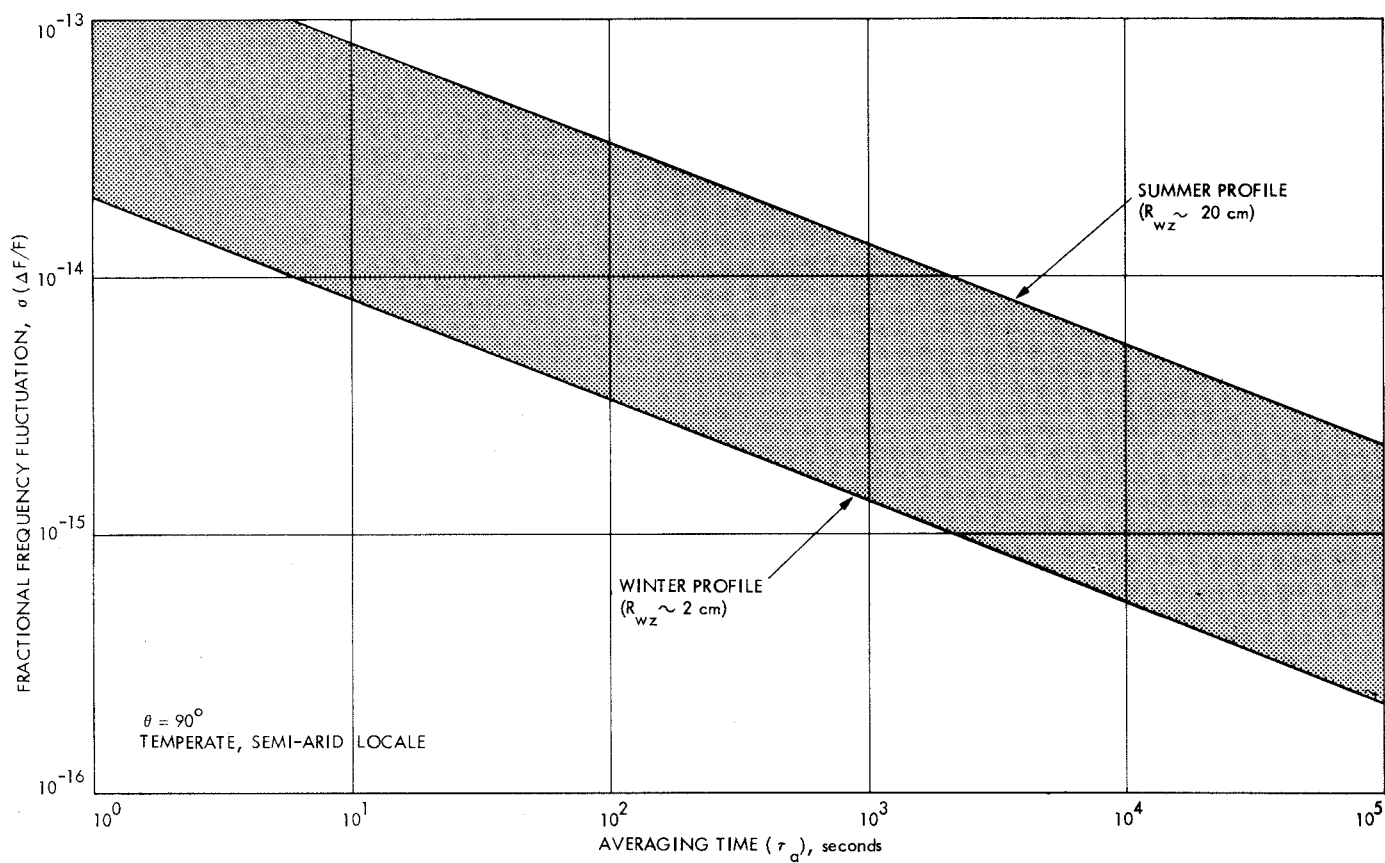
Knowledge of low frequency wet tropospheric fluctuation will be very important to the suggested use of ultra-precise Doppler data in the search for gravitational waves. By noting the similarities between the solar wind and the troposphere, one can hypothesize a parametric model for the tropospheric path length fluctuations. In addition, recent experimental observations of the wet tropospheric signal delay fluctuation spectrum can be interpreted as confirmation of the hypothesized model.

The hypothesized model immediately suggests certain bounding conditions under which experimental tropospheric fluctuation data should be acquired. These are as follows (chosen to maximize the expected parametric variations):

- (1) Climatic conditions
  - (a) Winter (cold and dry)
  - (b) Summer (warm and humid)
- (2) Elevation angle
  - (a) Zenith
  - (b) Near horizon
- (3) Averaging times
  - (a) 60 seconds
  - (b) 260,000 seconds

## References

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**Fig. 1. Wet tropospheric signal delay fluctuation versus averaging time**